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International Journal of Heat and Mass Transfer 49 (2006) 4687–4695
International Journal of HEAT and MASS TRANSFER
www.elsevier.com/locate/ijhmt

Technical Note

Integral transform solution for natural convection in three-dimensional porous cavities: Aspect ratio effects

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Received 4 November 2005; received in revised form 29 April 2006
Available online 21 July 2006

Abstract

Three-dimensional natural convection in box-like cavities filled with a porous material is revisited, by considering a transient formulation for the energy balance and a quasi-steady formulation for the flow problem. The Generalized Integral Transform Technique (GITT) is employed in the hybrid numerical-analytical solution of the Darcy law based model for vertical cavities (insulated vertical walls with differentially prescribed horizontal wall temperatures), employing the vorticity-vector potential formulation. Comparisons with previously reported numerical solutions are performed and the transition between conductive and convective states is illustrated, centering on the aspect ratio influence on the flow and heat transfer phenomena. A set of reference results for the steady-state behavior under different aspect ratio is provided for covalidation purposes.

Keywords: Natural convection; Transition; Porous media; Rectangular cavities; Integral transforms

1. Introduction

Due to the large number of applications involving buoyancy induced flows in saturated porous media in different processes, such as those in the chemical, mechanical, environmental and geological fields (e.g., heat, mass and fluid flow in fixed bed chemical reactors, filtering processes, geothermal systems and petroleum reservoirs, to name a few), the prediction of natural convection in porous media filled cavities has deserved a broad scope of publications and reviews along the last few decades [1,2]. Following the same trend as in all branches of the physical sciences, a considerable amount of computer simulation work has been devoted to this fundamental problem in thermal sciences, aimed at producing benchmark results for the validation of general purpose computer software. Such refined simulation tasks have generally been achieved with the aid of conventional numerical techniques, and most frequently for mathematical formulations concerning two-dimensional geometries under different flow models. For three-dimensional formulations, due to the sometimes prohibitive computational effort associated with discretization processes, the literature is less abundant [3–13]. The available works for this situation in general adopt the Darcy flow model, together with the assumptions of constant and isotropic physical properties and linear variation with temperature of the buoyancy term (Boussinesq approximation). In addition, the cubic geometry is the most frequently one considered, and the situation of a vertical enclosure (a heated base and thermally insulated vertical walls) is commonly employed as the test case for the covalidation of solution methodologies.

Numerical results from all such different research efforts are far from coincident, seldom available in tabular form, and quite rarely for the full transient situations. Quite

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doi:10.1016/j.ijheatmasstransfer.2006.04.028

Nomenclature

a, b	cavity width and depth, respectively
c_p	specific heat
d	cavity height
Da	Darcy number
g	gravity acceleration, $\boldsymbol{g} = (0, 0, -g)$
F_s	cavity shape factor
h_m	average heat transfer coefficient
K	permeability of solid matrix
k_m	thermal conductivity of porous medium, $k_m = \Phi k_f + (1-\Phi)k_s$
k_s	thermal conductivity of solid matrix
Mx, My	aspect ratio in x and y directions, respectively
Nu	Nusselt number, $Nu = h_m d/k_m$
$\overline{Nu}$, Nu_G	overall Nusselt number
Nu_C	characteristic overall Nusselt number
NT	truncation order of the eigenfunction expansion solution (number of terms)
Ra	modified Rayleigh number
t	time
T	temperature
V_C	cavity characteristic volume
(x^*, y^*, z^*)	space coordinates
(x, y, z)	dimensionless coordinates

Greek symbols

α_m	solid matrix thermal diffusivity – $\alpha_m = k_m/(\rho c_p)_{fluid}$
β	fluid thermal expansion coefficient
$\theta\ (x,y,z,\tau)$	dimensionless temperature
$\tilde{\tilde{\theta}}_{H_{imq}}(\tau)$	transformed temperature field
μ	fluid absolute viscosity
ν	fluid kinematic viscosity – $\nu = \mu/\rho$
ρ	fluid density
ρ_s	density of solid matrix
τ	dimensionless time
ψ	vector potential – ψ (ψ_x, ψ_y, ψ_z)
$\tilde{\tilde{\psi}}_{x_{imq}}$	transformed vector potential component in the x direction
$\tilde{\tilde{\psi}}_{y_{imq}}$	transformed vector potential component in the y direction

recently [14], the Generalized Integral Transform Technique (GITT) [15–17] has been employed in the development of a hybrid numerical–analytical solution for natural convection in the cubic cavity situation, allowing for a thorough comparison of previously published numerical results for the steady state Nusselt number. This hybrid approach has been demonstrated to be an efficient tool in the production of benchmark results in nonlinear diffusion and convection–diffusion problems and has been progressively advanced towards the automatic error-controlled solution of such partial differential problems. At this point, it is worth mentioning some illustrative contributions on this method for the specific class of problems of interest here, namely, the solution of natural convection problems in cavities under steady and transient regimen, for both porous media or just fluid filled two-dimensional enclosures [18–23]. The streamfunction-only formulation was preferred in all such contributions on natural convection because of the inherent advantages in its combined use with this hybrid approach, as more closely discussed in [15]. Later on, this hybrid solution scheme was advanced to handle the three-dimensional Navier–Stokes equations [24] based on the vector–scalar potentials formulation [25–27], with similar computational advantages with respect to the two-dimensional case. Since the pioneering work of Aziz and Hellums [25], the vorticity-vector potential approach has been receiving increasing attention, when it was shown that this formulation could lead to more stable and fast simulations of three-dimensional flows. This formulation was itself originally applied to three-dimensional natural convection in porous media [3].

The present contribution, following the efforts initiated in previous works that analyzed three-dimensional flows via integral transforms [14,24], is aimed at advancing this computational tool towards the accurate solution of natural convection within porous media filled rectangular three-dimensional cavities of arbitrary aspect ratio. We assume the Darcy flow model with the governing equations expressed in terms of the vorticity-vector potential formulation, considering a transient formulation for the energy balance and a quasi-steady formulation for the flow problem. The computer code was modified to produce reliable numerical results for the temperature field and Nusselt numbers for cavities of arbitrary aspect ratio, when reordering procedures and other computational aspects were emphasized in the improvement of the convergence behavior of the GITT approach, thus extending the algorithm constructed in [14]. The transition between conductive and convective states due to the variation of the cavity geometric parameters is also more closely examined.

2. Analysis

Three-dimensional natural convection in an impermeable box-like cavity filled with a porous material and saturated with a Newtonian fluid is considered. The flow is buoyancy induced by heat exchange between the fluid-porous media and the top and bottom walls.

The configuration considered (Fig. 1) is known as the vertical cavity problem, in which boundary conditions of first kind, T_0 and $T_0 + \Delta T$, respectively, are imposed at the top and at the bottom walls, whilst the vertical walls

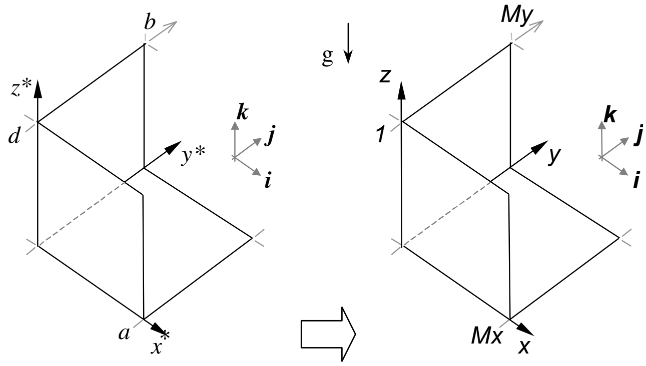

Fig. 1. Geometry and coordinates systems for natural convection in a three-dimensional porous cavity.

are insulated. The governing dimensionless equations for this problem, in terms of the vorticity-vector potential dependent variables, within the validity of Darcy's model, and after invoking the Boussinesq approximation, are given by Eqs. (1)–(5) below [14,28]. However, we should note that the transient term in the momentum equations was neglected, as usual in dealing with stable Darcian flows. It has also been shown in [29] that for typical values of the associated parameters, the order of the transient term coefficient is at most $O(10^{-4})$. Thus, the employed dimensionless formulation is written as:

Momentum equations:

x-component:

$$Ra\frac{\partial\theta}{\partial y}+\nabla^2\psi_x=0,\qquad 0<x<Mx,\quad 0<y<My,\quad 0<z<1,\quad \tau>0 \tag{1}$$

y-component:

$$-Ra\frac{\partial\theta}{\partial x}+\nabla^2\psi_y=0,\qquad 0<x<Mx,\quad 0<y<My,\quad 0<z<1,\quad \tau>0 \tag{2}$$

Energy equation:

$$\frac{\partial\theta}{\partial\tau}-\frac{\partial\theta}{\partial x}\frac{\partial\psi_y}{\partial z}+\frac{\partial\theta}{\partial y}\frac{\partial\psi_x}{\partial z}+\frac{\partial\theta}{\partial z}\left[\frac{\partial\psi_y}{\partial x}-\frac{\partial\psi_x}{\partial y}\right]=\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}+\frac{\partial^2\theta}{\partial z^2},\qquad 0<x<Mx,\quad 0<y<My,\quad 0<z<1,\quad \tau>0 \tag{3}$$

Boundary conditions:

$$x=0\rightarrow\partial\psi_x/\partial x=\psi_y=0,\qquad \partial\theta/\partial x=0,\quad 0<y<My,\quad 0<z<1 \tag{4a–c}$$

$$x=Mx\rightarrow\partial\psi_x/\partial x=\psi_y=0,\qquad \partial\theta/\partial x=0,\quad 0<y<My,\quad 0<z<1 \tag{4d–f}$$

$$y=0\rightarrow\partial\psi_y/\partial y=\psi_x=0,\qquad \partial\theta/\partial y=0,\quad 0<x<Mx,\quad 0<z<1 \tag{4g–i}$$

$$y=My\rightarrow\partial\psi_y/\partial y=\psi_x=0,\qquad \partial\theta/\partial y=0,\quad 0<x<Mx,\quad 0<z<1 \tag{4j–l}$$

$$z=0\rightarrow\psi_x=\psi_y=0,\theta=1,\qquad 0<x<Mx,\quad 0<y<My \tag{4m–o}$$

$$z=1\rightarrow\psi_x=\psi_y=0,\theta=0,\qquad 0<x<Mx,\quad 0<y<My \tag{4p–r}$$

Initial condition:

$$\tau=0\rightarrow\theta=0, 0<x<Mx,\qquad 0<y<My,\quad 0<z<1 \tag{5}$$

where the following dimensionless groups were employed

$$x=x^*/d,\quad y=y^*/d,\quad z=z^*/d,\quad Mx=a/d,\quad My=b/d,\quad \theta=(T-T_0)/\Delta T,\quad \tau=k_{\mathrm{m}}t/[(\rho c_p)_{\mathrm{m}}d^2],\quad Da=K/d^2,\quad Ra=\beta\Delta TgKd/(\nu\alpha_{\mathrm{m}}) \tag{6a–j}$$

in which, T_0 and T_R are reference temperatures, Da represents the Darcy number, and Ra is the modified Rayleigh number, both for the porous medium. It should be noted that the geometry and boundary conditions of the considered example, result in a vanishing z-component of the vector potential, i.e., the process of decomposition for this component results in $\nabla^2\psi_z=0$, together with $\psi_z=0$ at $x=0$, Mx; $y=0$, My and $\partial\psi_z/\partial z=0$ at $z=0$ and 1, leading to the trivial solution $\psi_z(x,y,z)=0$, $\forall x, y, z$, $0\leqslant x\leqslant Mx$, $0\leqslant y\leqslant My$, $0\leqslant z\leqslant 1$.

The solution methodology here employed is the GITT approach [15–17], and the details of its application for a system of equations such as the present can be found in references [14,28,30]. Thus, the solution for the components of the vector potential and the temperature field, as well as other quantities of practical interest, are then readily obtained in the form:

Table 1
Steady state overall Nusselt number for different values of aspect ratio and Rayleigh number

Ra	Mx	My	Nu_G [a]	Regime [d,a]	Nu_G [b]	Regime [d,b]	Dif
60	1	1	1.72014	3D–1C	1.67	3D–1C	−2.9%
60	1	0.5	1.79636	2D–1C/XZ	1.89	2D–1C/XZ	+5.2%
100	0.5	0.5	2.13630	2D–1C/XZ	–	–	–
100	0.5	1.0	2.64594	2D–1C/YZ			
100	1	1	2.64592	2D–1C/YZ	–	–	–
100	1	2	2.58523	2D–3C/YZ	–	–	–
100	1	4	2.62690	2D–5C/YZ	–	–	–
100	1	8	2.64101	2D–7C/YZ	–	–	–
120	1	1	2.94906	2D–1C/XZ	3.49	2D–1C/XZ	−18.3%[(2D)]
					3.94	3D–1C [c]	
120	3	1	3.00151	2D–4C/XZ	3.49	2D–3C/XZ	−16.3%[(2D)]
					3.57	3D–1C [c]	
120	3	0.5	2.99683	2D–4C/XZ	3.49	2D–3C/XZ [c]	−16.5%[(2D)]
					2.98	3D–1C	

Dif – Relative difference from present results.
[a] Present.
[b] Holst and Aziz [3].
[c] Pointed as the physically preferred solution, based on Platzman criterion [31]).
[d] Regime:two-dimensional (2D) or three-dimensional (3D), number of cells and convection plane for 2D flow.

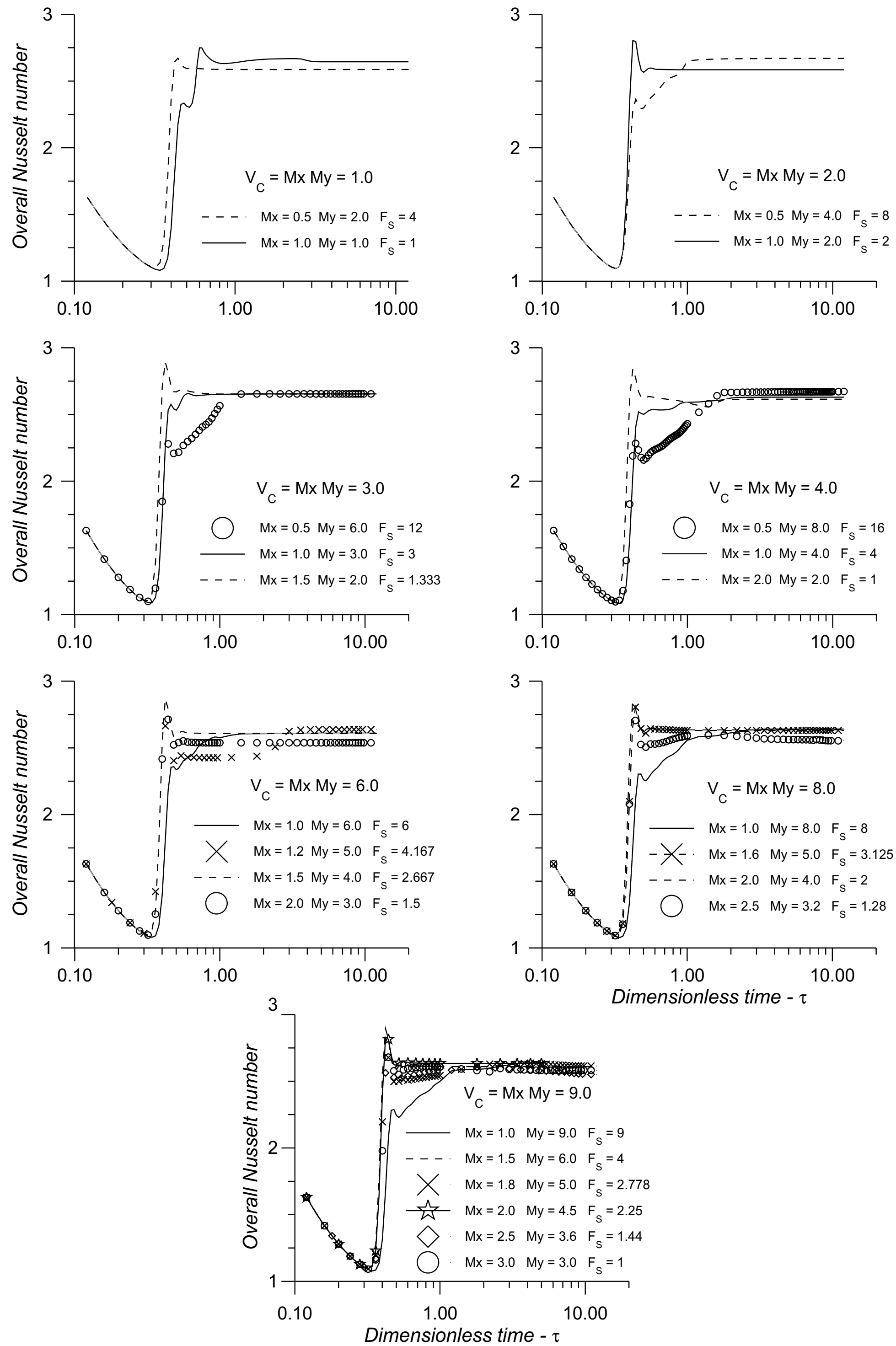

Fig. 2. Influence of shape factor on the transition behavior of the overall Nusselt number ($Ra = 100$).

$$\psi_x(x,y,z) = -Ra\sum_{\hat{i}=1}^{NT} \frac{\varepsilon_m(\hat{i})}{\hat{\alpha}_{\hat{i}}^2} A_{i(\hat{i})}(x) B_{m(\hat{i})}(y) C_{q(\hat{i})}(z) \bar{\tilde{\theta}}_{H\hat{i}}(\tau) \tag{7a}$$

$$\psi_y(x,y,z) = Ra\sum_{\hat{i}=1}^{NT} \frac{\alpha_i(\hat{i})}{\hat{\alpha}_{\hat{i}}^2} D_{i(\hat{i})}(x) E_{m(\hat{i})}(y) C_{q(\hat{i})}(z) \bar{\tilde{\theta}}_{H\hat{i}}(\tau) \tag{7b}$$

$$\theta(x,y,z,\tau) = (1-z) + \sum_{\hat{i}=1}^{NT} A_{i(\hat{i})}(x) E_{m(\hat{i})}(y) C_{q(\hat{i})}(z) \bar{\tilde{\theta}}_{H\hat{i}}(\tau) \tag{7c}$$

where all quantities that appear in the summations above are exactly those found in reference [14]. From the definition for the average Nusselt number we obtain:

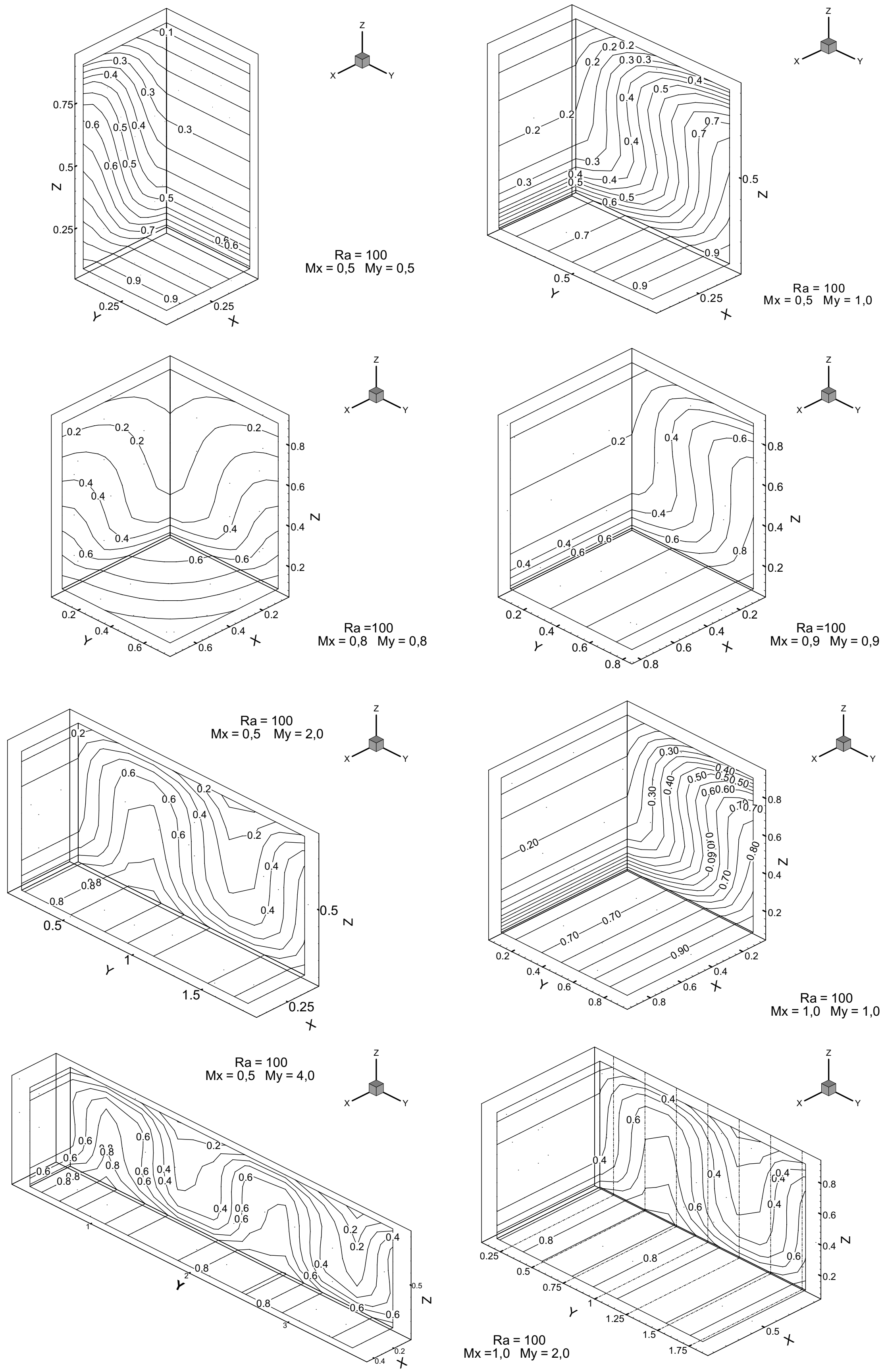

Fig. 3a. Steady state temperature isolines close to the rear faces of the cavities for characteristic volumes varying from 0.25 to 2.0.

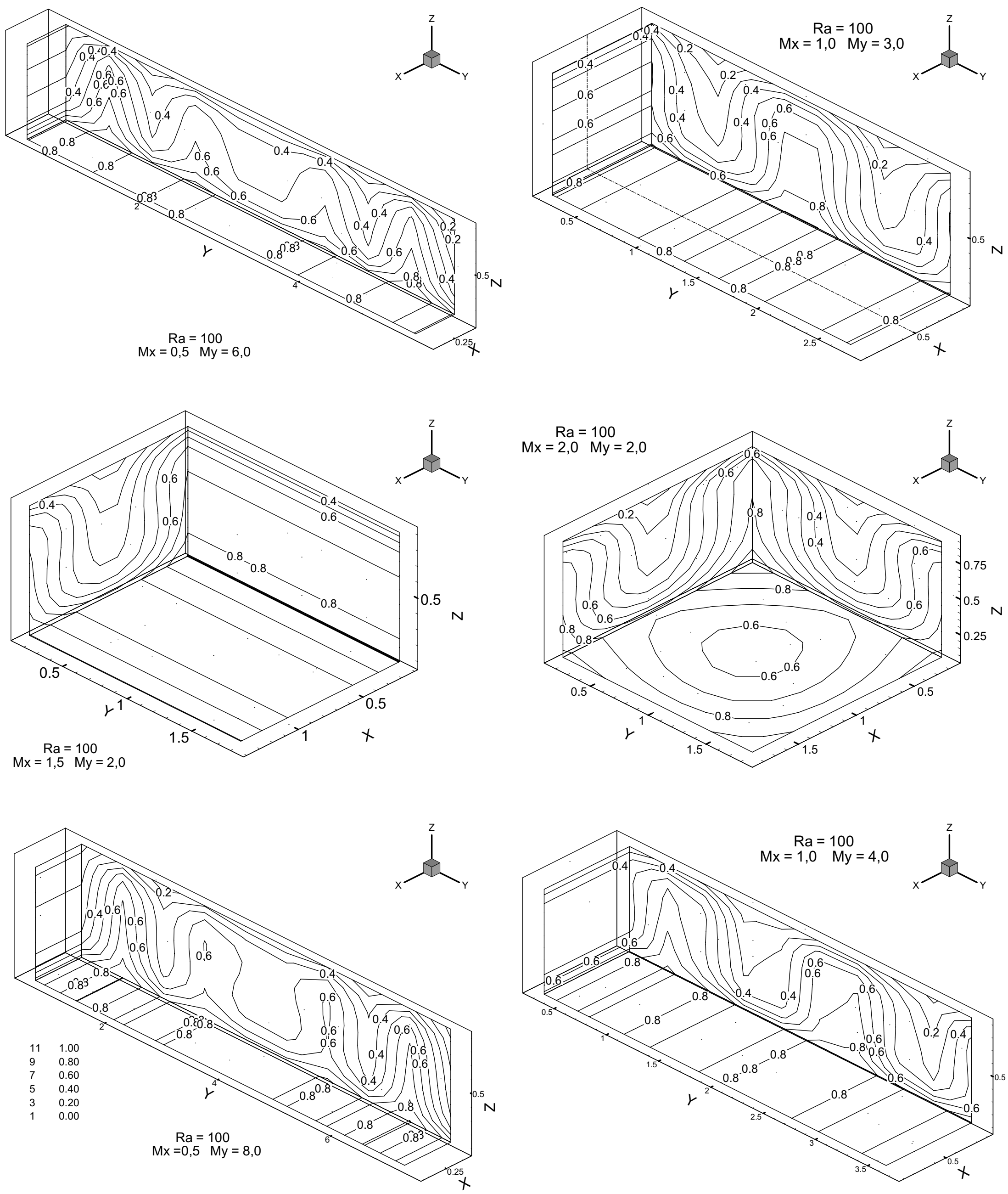

Fig. 3b. Steady state temperature isolines close to the rear faces of the cavities for characteristic volumes varying from 3.0 to 4.0.

$$\overline{Nu}(\tau)\big|_{z=\breve{z}} = \frac{-1}{MxMy}\int_0^{Mx}\int_0^{My}\frac{\partial\theta}{\partial z}\bigg|_{z=\breve{z}}\,\mathrm{d}y\,\mathrm{d}x = 1-\frac{1}{MxMy}\sum_{\hat{i}=1}^{\mathrm{NT}}\left[\int_0^{Mx}A_{i(\hat{i})}(x)\,\mathrm{d}x\int_0^{My}E_{m(\hat{i})}(y)\,\mathrm{d}y \times \frac{\mathrm{d}c_{q(\hat{i})}(z)}{\mathrm{d}z}\bigg|_{z=\breve{z}}\,\overline{\overline{\tilde{\theta}}}_{H\hat{i}}(\tau)\right] \quad (8)$$

For the analysis here presented the following definitions are also introduced: characteristic cavity volume; characteristic overall Nusselt number (the overall Nusselt number Nu_G scaled with the characteristic cavity volume, here used for a better visualization of the different curves associated with the various cavities) and cavity shape factor (the ratio of the cavity aspect ratios), which are respectively given as

$$V_C = Mx.My.1; \qquad Nu_C = V_C\overline{Nu}(\tau)\big|_{z=\breve{Z}} = V_C Nu_G; \qquad F_S = My/Mx \quad (9\text{–}11)$$

3. Results and discussion

The literature review confirms the lack of reference results for natural convection in porous three-dimensional enclosures for non-unitary aspect ratios [32]. Among the very few tabulated results, we must refer to the pioneering

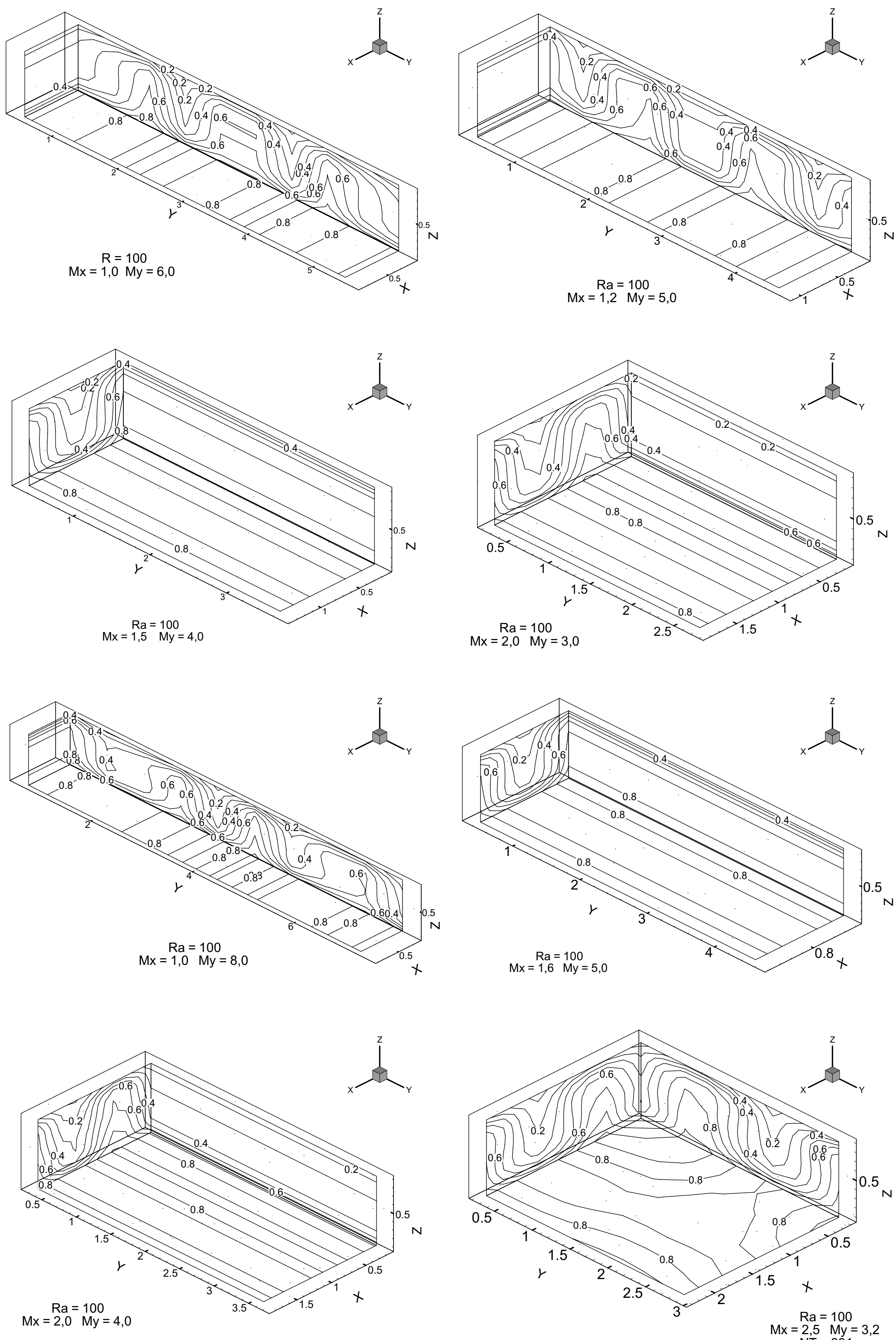

Fig. 3c. Steady state temperature isolines close the rear faces of the cavities for characteristic volumes varying from 6.0 to 8.0.

work of Holst and Aziz [3], who several decades ago presented values for the steady state overall Nusselt number. As shown in Table 1, these results are in relative good agreement with those here obtained, which have already been covalidated against various other more recent works for the cubic cavity case [14]. For $Ra = 60$, the differences on the overall Nusselt number are only around 5%, for the same final convection regime, while for $Ra = 120$ these differences reach 18%, and different final convection regimen for the cases of a cubic cavity and a parallelepiped with $Mx = 3$ and $My = 1$. Such deviations are not so relevant considering the pioneering numerical work in Ref. [3], obtained more than thirty years ago with a fully discrete approach and limited computational resources. Some extra results for $Ra = 100$ are also added in Table 1, under different aspect ratios, for reference purposes.

As pointed out before [29], application of the classical Darcy model to the vertical cavity problem above described, directly leads to pure conduction, since the elimination of the transient term in the flow equations tends to shift the onset of the convection regime to infinity. For the convection regime to be triggered, a perturbation has to be introduced in the governing equations. In this way, the quasi-steady Darcy flow model cannot in fact predict the actual transient behavior in the present physical situation, but rather offers information on the qualitative behavior and reproduces the desired steady state solution, after the convection regime is triggered by the initial condition perturbation [30]. In the following analysis, a value of 10^{-6} in the initial condition perturbation was employed throughout.

For a better understanding of the aspect ratio effect on the transition process, Fig. 2 shows the evolution of the overall Nusselt number for cavities with different characteristic volumes. It can be observed that the transition from the conductive to the convective regime takes place through distinct behaviors, according to the shape factor, Eq. (11). Cavities with higher shape factor present a wider transition regime, associated with the formation, movement and accommodation of convective cells. For the cases here considered, it can also be observed that for cavities with shape factor larger than 6, the transition regime is not only wider, but also without a peak in the overall Nusselt number which is present in the other situations. The influence of the shape factor on the transient regime is better observed on the different plots of Fig. 2, where it is also presented the evolution of the overall Nusselt number for various cavities with the same characteristic volumes. As expected, for the same value of Rayleigh number, the convective process prevails over the conductive one within a fairly narrow dimensionless time range, from 0.3 to 0.35, for all cavities, roughly independent of characteristic volume and shape factor, corresponding to the necessary time for the diffusion front to more noticeably reach the cavity top.

With respect to the evolution of the convective processes, for the cases in which the steady state has been achieved within the range of the dimensionless time considered, in only one case, $Mx = 0.8$ and $My = 0.8$, a three-dimensional convective pattern was observed. In all other situations the stable regime was observed to occur under a two-dimensional pattern, with flow characterized by the formation of one to eight convective cells. Figs. 3a–3c illustrate the steady state temperature behavior close to the rear faces of the cavities, for some of the cases studied, from which one may identify the convective regimen pointed out in Table 1.

4. Conclusions

Natural convection in laminar regime inside three-dimensional box-like cavities filled with porous material is analyzed, by considering a transient formulation for the energy balance and a quasi-steady formulation for the flow problem. The Generalized Integral Transform Technique (GITT) is employed, in the form of a hybrid numerical-analytical solution, with explicit analytical expressions for the spatial behavior of the vector potential components and temperature and numerical computation for the transformed time dependent temperature. A set of reference results for the overall Nusselt numbers in the steady state with different geometric configurations is established. In addition, the transition behavior of the convection phenomena within typical cavities is graphically illustrated, with emphasis on the identification of the aspect ratio influence.

Acknowledgements

The authors would like to acknowledge the financial support provided by PRONEX, CNPq, and FAPERJ, all of them sponsoring agencies in Brazil.

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